Another Algebra I Supplement Review

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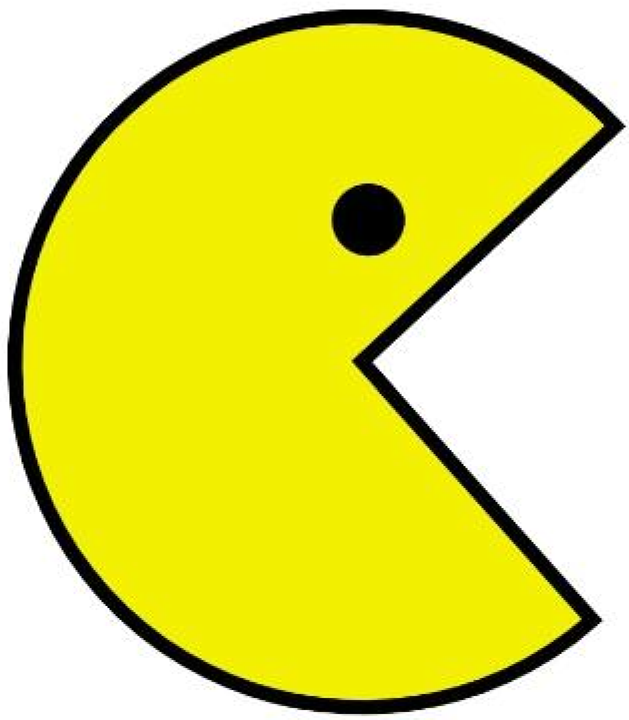
Solving Inequalities:



To where does he devour?

Comparison-based Relational Symbols

Greater than: **>**

Less than: **<**

Greater than or equal to: **≥**

Less than or equal to: **≤**

Solve and Graph the Inequality - Practice

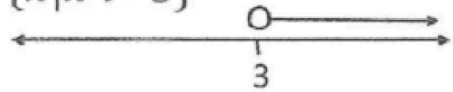
1) $3 < x$

2) $x \geq 4$

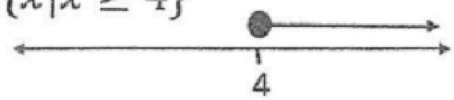
3) $\frac{x}{7} \geq -6$

4) $-\frac{x}{2} \geq 3$

1) $\{x | x > 3\}$

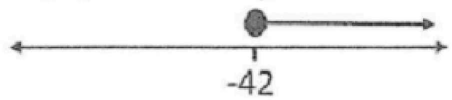


2) $\{x | x \geq 4\}$



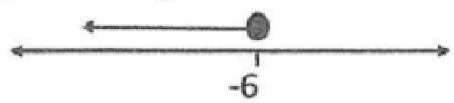
3) $\frac{7}{1} \bullet \frac{x}{7} \geq -6 \bullet \frac{7}{1}$

$\{x | x \geq -42\}$



4) $-\frac{x}{2} \bullet -\frac{2}{1} \leq 3 \bullet -\frac{2}{1}$

$\{x | x \leq -6\}$



Can you graph an example that uses the missing symbol from above?

Consider the subtle and slight difference between parentheses and square brackets.

Solve $2x + 10 \geq 7(x + 1)$. State answer in interval notation.

$$\left(-\infty, \frac{3}{5}\right]$$

Solve $-x + 6 > -(2x + 4)$. State answer in interval notation.

$$(-10, \infty)$$

How to express and state your answer: Interval Notation vs. Set Notation

Example 1:

Let our solution set for x be defined as: $x > 3$.

You can express this idea in interval notation as: $(3, \infty)$.

-or-

You can express this idea in set notation as: $\{ x \mid x > 3 \}$.
Read as, “The *set* of x, such that x is greater than 3” .

When working with inequalities:

1) **Pretend it is just an equation symbol at first and solve accordingly.**

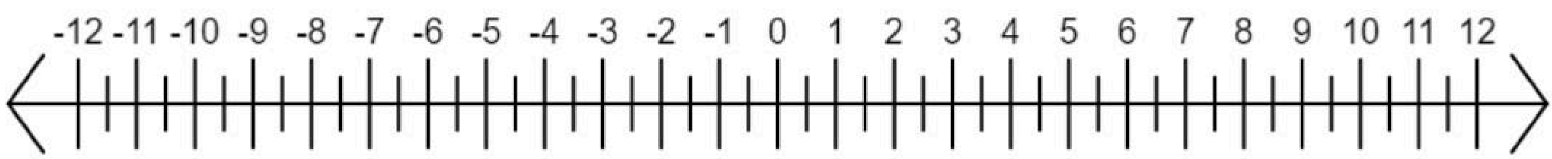
2) **If you multiply or divide by a negative value, reverse the direction of “packman’s mouth.” That is reverse the direction of the inequality.**

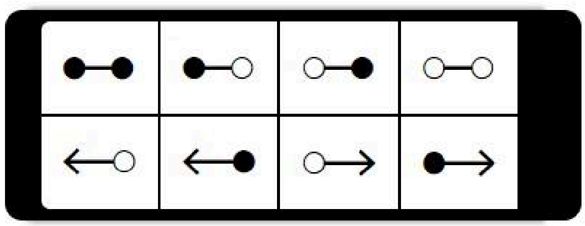
3) **When graphing for $<$ or $\leq$, graph as usual but sketch below the line. For $>$ or $\geq$, sketch above. Finally for $<$ or $>$ make the line dotted, otherwise it is solid.**

Question 1:

Solve $4 - \frac{2}{3}x > 2 - x$ for x. Plot the solution set on the number line.

Select a solution set indicator. Then, select the number line and drag the point(s) to appropriate location(s).





Question 2:

A) Let $10x - 5 > 7$; Solve for x.

B) Let $-10x - 5 > 7$; Solve for x.

C) Graph the first inequality.

D) Graph the second inequality.

E) Let $y > x$; graph the bi-variable inequality.

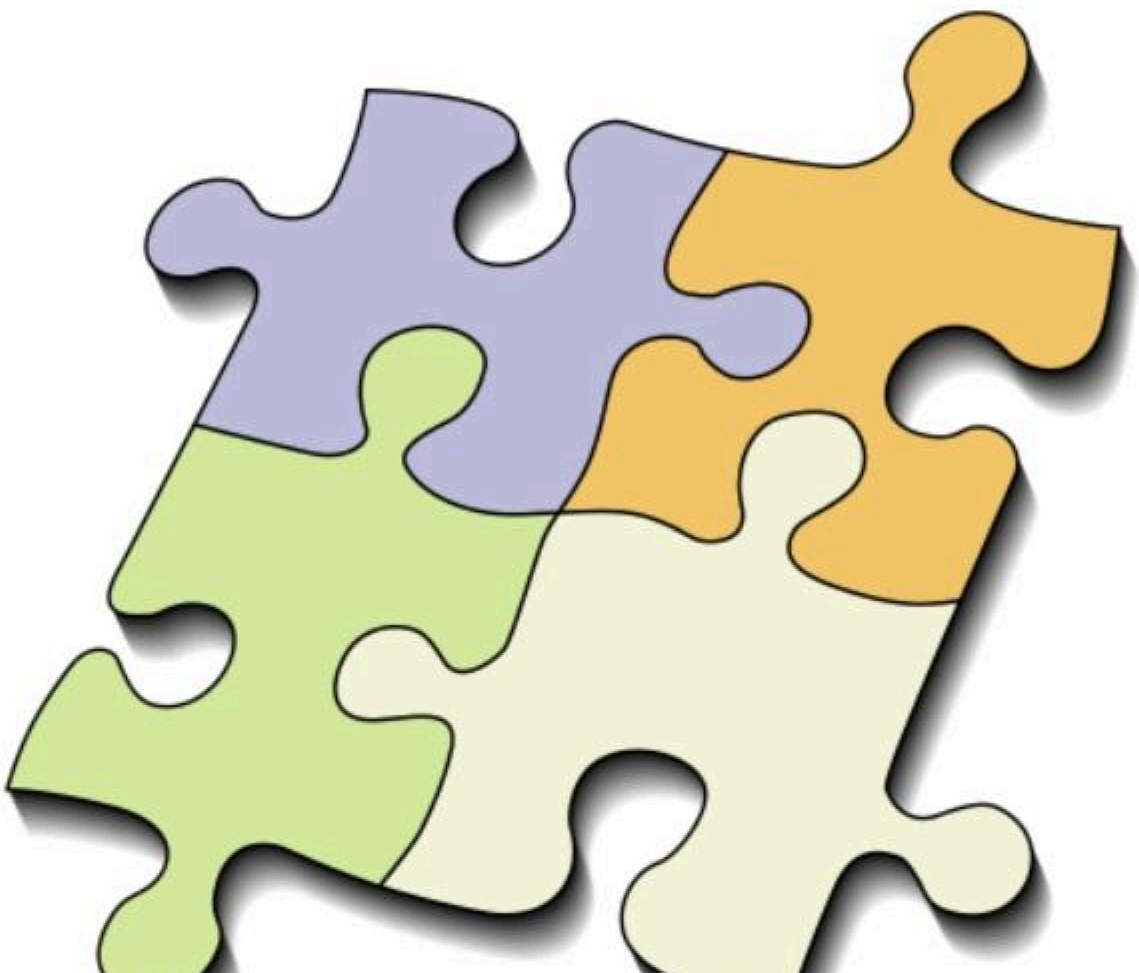
F) Let $y \leq -2x + 7$; graph please.

G) Let $y \geq x^2 - x - 1$; graph please.

H) Let $y < x$ and $y \geq x^2$ on the interval x in $[0, 1]$, graph please.

I) Let $5y + x < 7$ and $-y \leq x^2 + (½)$, graph please.

Piecewise functions:

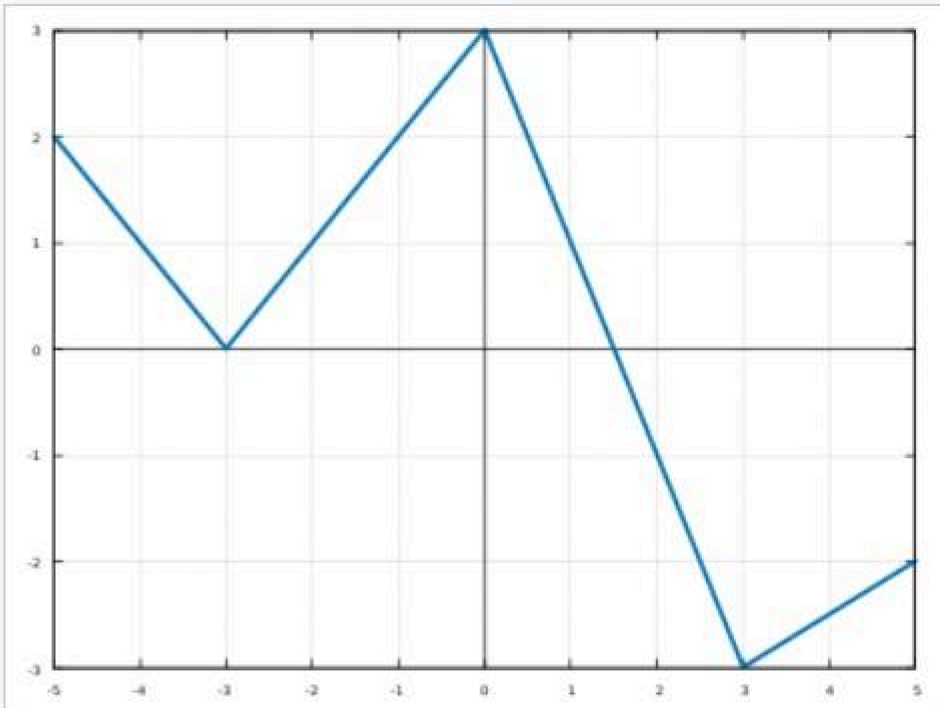


A piecewise function is a function that is defined as an amalgamation of pieces.

For example:

Plot of the piecewise linear function

$$f(x) = \begin{cases} -3 - x & \text{if} \quad x \leq -3 \\ x + 3 & \text{if} \quad -3 \leq x \leq 0 \\ 3 - 2x & \text{if} \quad 0 \leq x \leq 3 \\ 0.5x - 4.5 & \text{if} \quad 3 \leq x \end{cases}$$



Plot of the piecewise linear function

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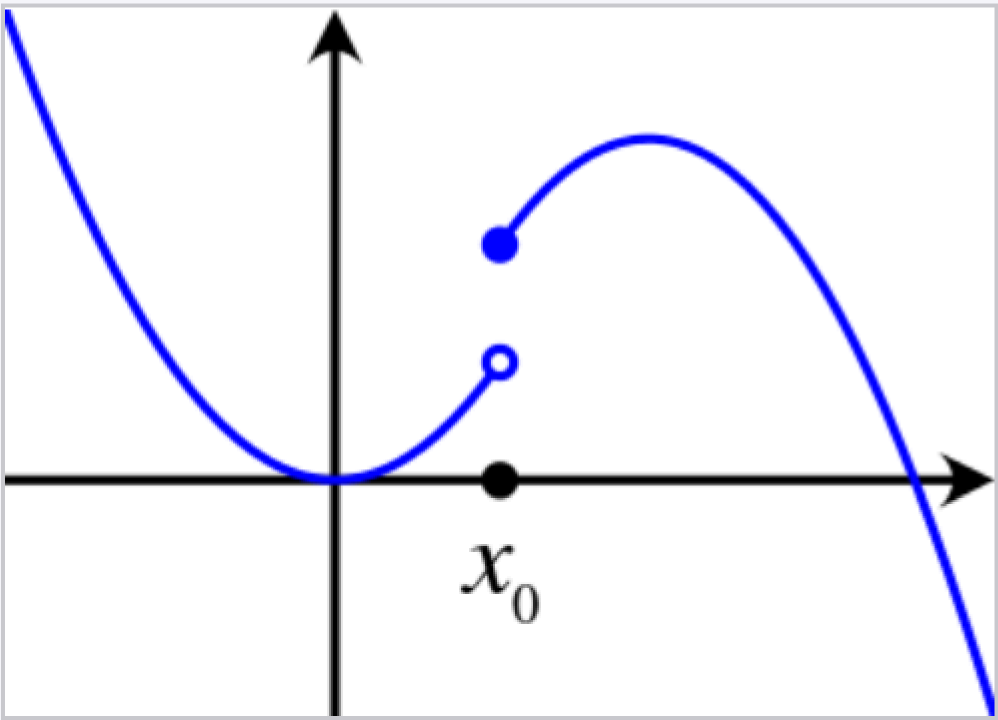
Notice how each interval of x produces a different "piece" of f(x).

A second example:



A piecewise function comprising different quadratic functions on either side of x_0.

Question 3:

graph :

$$f(x) = \begin{cases} x & , -3 < x \leq 5 \\ x^2 + 6 & , 5 < x \end{cases}$$

Exponents and Exponential Functions

For complex z

$$e^z = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{z^n}{n!}.$$

If this is bewildering, don't worry, the full details of the above power series is beyond this course.

However, since 1 is an element of the complex numbers, you may plug in 1 for z and use a calculator to approximate e in your spare time.

(Hint: 3! = 3*2*1).

The above image is also from wikipedia.[1]

[1] https://en.wikipedia.org/wiki/Euler%27s_formula

Examples to demonstrate rules for exponents:

$5^2 = 5*5 = 25$.

$5^{-2} = 1/25 = 0.04$.

$(5^2)^2 = (25)^2 = 625 = 5^4 = (5^2)(5^2)$.

$25^{(1/2)} = 25^{0.5} = 5$. (another way to write square root)

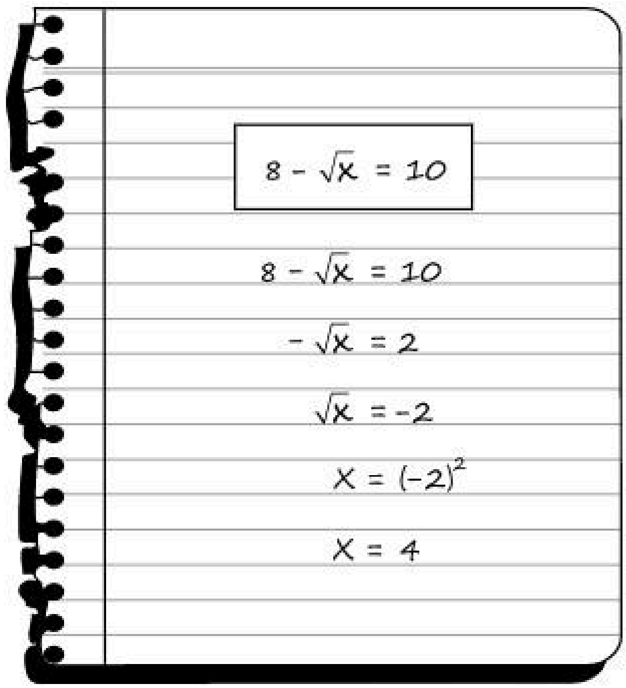
$e^{nl\ 5} = 5$. (read: “e raised to the natural log base e of 5 equals 5.)

Note that the natural log is the inverse operation of exponentiation. It spits out an exponent. e raised to an exponent, let’s call it “banana”, equals 5. So e raised to the “banana” is again 5.

$5^0 = 1 = (5^2)(5^{-2})$.

Question 4:

Mark solved the equation in the box, using the steps shown.



Is the solution $x = 4$ correct? State yes or no, and justify your answer.

Question 5:

Which value is a solution of the equation $x^2 = \frac{2x}{x+1}$?

Select **all** that apply.

- ☐ A. $x = {}^{-}2$
- ☐ B. $x = {}^{-}1$
- ☐ C. $x = 0$
- ☐ D. $x = 1$
- ☐ E. $x = 2$

Polynomials:

For our purposes, we will stay in 2 dimensions.

Consider the *Fundamental Theorem of Algebra:*

A polynomial of degree d, with real or complex *coefficients*, has <u>at most</u> d distinct *roots* (some in *R*, and all in *C* since *R* $\subset$ *C*.)

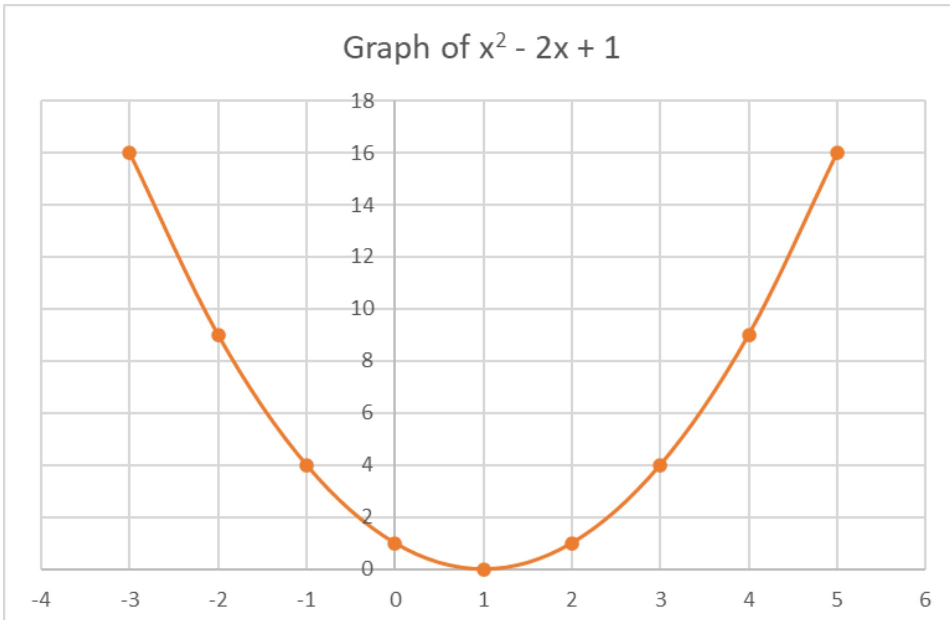
For example: $P(x) = x^2 - 2x + 1$ has 2 roots $x = 1$ and $x = 1$.

***Proof:* Let $x^2 - 2x + 1 = 0$,**

so then $(x - 1)*(x - 1) = 0$.

So $x = 1$ is our root, and we have two copies basically.

Now the above polynomial can be graphed as follows:



Notice our root (the point where the curve hits the x-axis) is at (1, 0). Notice the zero in the y coordinate slot. That's why we set the polynomial equal to zero and solve for our x coordinates. The x coordinates that we find are called the roots of our polynomial.

A *Quadratic equation* is of degree 2. The highest power of any of the variables is 2.

For example:

$$x^2 - x - 1 .$$

A Cubic equation is of degree 3…

$$ax^3 + bx^2 + cx + d .$$

Factoring Polynomials
and finding the roots

The _Quadratic Formula_ is a sure fire way to find the roots of a quadratic equation.

For example: $x^2 - x - 1$.

Using the quadratic formula:

$$\frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Notice that our _discriminant_ (b^2 - 4ac) is entirely under the _vinculum_, which is our square root bar.

This formula also has two parts. The first is -b plus the square root. The second is -b minus the square root.

Note: The discriminant is different from the determinant, which we will cover later with matrices.

Plugging in accordingly from the example above:

$$\frac{1-\sqrt{(}5)}{2}$$

$$\frac{1+\sqrt{(}5)}{2}$$

Notice that we have two answers, two roots for our quadratic equation of degree 2.

Plug into a calculator and we will see how these roots converge to the *golden ratio*.

For trivial quadratic equations, like:

$$x^2 + 6x + 5 .$$

We can simply guess and check.

$$x^2 + 6x + 5 = (x+1)(x+5) = 0 .$$

So for this example, our roots are x = -1 , & x = -5 .

Question 6:

Consider the equation $p^2 - 5p - 6 - x(p-6)^2 = 0$, where p is a real constant.

Part A

If $p = 6$, then the equation has

- A. no real solutions.
- B. exactly one real solution.
- C. exactly two real solutions.
- D. infinitely many real solutions.

Part B

If $p \neq 6$, then $x =$

- A. $\frac{p-2}{p-6}$
- B. $\frac{p-1}{p-6}$
- C. $\frac{p+1}{p-6}$
- D. $\frac{p+2}{p-6}$

Question 7:

Part A

Diane owns a store that sells computers. Her profit, in dollars, is represented by the function $P(x) = x^3 - 22x^2 - 240x$, where x is the number of computers sold.

Diane hopes to make a profit of at least $10,000 by the time she sells 36 computers. Explain whether or not Diane will meet her goal. Justify your reasoning.

Enter your explanation and your justification in the space provided.

- Math symbols
- Relations
- Geometry
- Groups
- Trigonometry
- Statistics
- Greek

Part B

Diane states that there are three possible values of x for which she will have a profit of $0. Find the values of x that produce a zero profit to show whether Diane is correct or not. Justify your reasoning.

Enter your answer and your justification in the space provided.

Question 8:

Which expression is equivalent to $3+2(x+4)(x-4)$?

- A. $2x^2-13$
- B. $2x^2-29$
- C. $2x^2-35$
- D. $5x^2-80$